

Effect of tilt on strong motion data processing

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Abstract

In the near-field of an earthquake the effects of the rotational components of ground motion may not be negligible compared to the effects of translational motions. Analyses of the equations of motion of horizontal and vertical pendulums show that horizontal sensors are sensitive not only to translational motion but also to tilts. Ignoring this tilt sensitivity may produce unreliable results, especially in calculations of permanent displacements and long-period calculations. In contrast to horizontal sensors, vertical sensors do not have these limitations, since they are less sensitive to tilts. In general, only six-component systems measuring rotations and accelerations, or three-component systems similar to systems used in inertial navigation assuring purely translational motion of accelerometers can be used to calculate residual displacements.

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1. Introduction

Strong-motion data are used in seismological studies and in earthquake engineering. For many years, from its birth in the 1930s, strong-motion seismology was mostly oriented toward earthquake engineering with very little impact on seismology. On the other hand, strong-motion also means that records are obtained in the near-field of an earthquake or an explosion, and therefore the seismologists started using near-field strong-motion records as a tool to study the earthquake source process. Compared to teleseismic records, near-field data present an opportunity to take a close look at a seismic source with much less distortion by the wave propagation path. Source studies also lead investigators to look at the possibilities of extracting more information from the records than what is possible with classical strong-motion data processing [1–3]. New methods of data processing were developed, allowing determination of ground motion including residual displacement from accelerograms [4–7].

Strong-motion seismometry employed the same pendulum type instruments as used in classical seismology. The main differences between strong-motion and weak-motion seismometers are that: (1) strong-motion instruments are less sensitive to the ground motion, and (2) their output is proportional to the acceleration as opposed to velocity or displacement in classical seismology. During 1930s the assumptions used in classical weak-motion seismology were simply transposed onto the area of strong-motion. The most questionable perception brought from classical seismology into strong-motion is the assumption of simple linear input motion of the ground, with rotational (tilt) component being negligible. These assumptions lead to two consequences: after 70 years of recording strong motion: (1) we still have very primitive knowledge about this important component of strong ground motion, with only theoretical or indirect assessments about the rotational components [8–13]. (2) We approximate the output of the instruments as translational acceleration. As a result, by integrating this signal we have to assume that this results in translational velocity and displacement.

In real near-field of an earthquake, the rotational components may not be negligible as compared to

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accelerations of the linear motion. As a result, the records that are assumed to represent translational accelerations are actually a sum of acceleration and tilt (it is well-known that the same type of pendulums can also be used as tiltmeters in the low frequency range).

Evidently, in the near-field of an earthquake it is necessary to measure all six components of the motion: three linear and three rotational. There are different ways of implementing this [14–21]. The general solution is to combine the three linear motion sensors with the corresponding three rotational ones at the same point of measurement. Another way is to assure that the seismic sensors are moving strictly linearly in space, for example, by using a gyroscopic platform (similar to the inertial navigation).

The need to consider rotational motions becomes especially important now because of the new trends in technology and data processing. Recently developed high-resolution digital accelerographs provide new possibilities for data processing, but must be applied carefully, and with full understanding of the basics and the possible errors in data recording. Different groups of researchers apply various techniques of acceleration data processing including permanent displacement calculations, but this can be done only if certain conditions apply.

2. Theory of the pendulum

Looking at the basic equation of pendulum motion one can discover the following interesting fact: it is written differently in different classical seismological sources [14,22,23].

The differential equation of a horizontal pendulum oscillating in a horizontal plane can be written as

(Fig. 1)

$$L : y_1'' + 2\omega_1 D_1 y_1' + \omega_1^2 y_1 = -x_1'' + g\psi_2 - \psi_3'' l_1 + x_2'' \theta_1 \tag{1a}$$

$$T : y_2'' + 2\omega_2 D_2 y_2' + \omega_2^2 y_2 = -x_2'' + g\psi_1 - \psi_3'' l_2 + x_1'' \theta_2 \tag{1b}$$

where

- y_i is the recorded response of the instrument,
- θ_i is the angle of pendulum rotation,
- l_i is the length of pendulum arm,
- $y_i = \theta_i l_i$,
- ω_i and D_i are, respectively, the natural frequency and fraction of critical damping of the i th transducer,
- g is acceleration due to gravity,
- x_i'' is the ground acceleration in i th direction,
- ψ_i is a rotation of the ground surface about x_i axis.

Eqs. (1a) and (1b) for the two horizontal directions L (longitudinal) and T (transverse) describe the pendulum response to low amplitude motions when $\sin(\psi_i) \approx \psi_i$. Fig. 1 shows a schematic representation of an accelerograph.

Sensitivity of the vertical pendulum to tilts is different. For small tilts it is proportional to

$$(1 - \cos(\psi)) \text{ and } \cos(\psi) \approx 1 - \psi^2/2$$

The equation of the vertical pendulum then can be written as follows:

$$V : y_3'' + 2\omega_3 D_3 y_3' + \omega_3^2 y_3 = -x_3'' + g\psi_1^2/2 - \psi_1'' l_3 + x_2'' \theta_3$$

Thus, the vertical pendulum is sensitive to the vertical acceleration, angular acceleration, and cross-axis motion, but is less sensitive to tilts (for small tilts).

Neglecting $g\psi_1^2/2$ gives:

$$V : y_3'' + 2\omega_3 D_3 y_3' + \omega_3^2 y_3 = -x_3'' - \psi_1'' l_3 - x_2'' \theta_3 \tag{1c}$$

Thus, a horizontal pendulum (1a) or (1b) is sensitive to the acceleration of linear motion, tilt, and angular acceleration, and cross-axis excitations. Regrettably, the completeness of representing Eqs. (1a)–(1c) in the literature varies. For example, Golitsyn [14] does not take into account the cross-axis sensitivity, while Aki and Richards [23] ignore the angular acceleration term.

It is important to study the sensitivity of a pendulum to the second, third and fourth terms on the right hand side of Eqs. (1a)–(1c). In teleseismic studies using typical seismometers, the effect of these terms is usually considered to be small enough to be neglected. The question is then: Is this also true for the strong-motion in the near-field studies? If the answer is ‘No’, then which terms on the right side of the equations will influence the output of the strong-motion instrument?

Possible impacts of different terms in the right hand side of Eqs. (1a)–(1c) were studied by Graizer [19], Trifunac, Wong

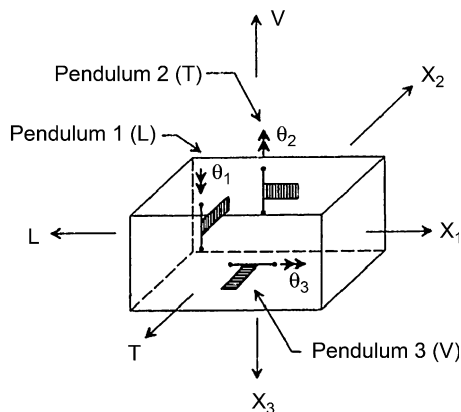


Fig. 1. Schematic representation of three transducers in an accelerograph. The coordinate axes X_1 , X_2 and X_3 serve to describe the motion of the L , T and V transducers, respectively. Angles θ_1 , θ_2 and θ_3 describe the deflection of the transducer pendulums (modified from Trifunac and Todorovska [24]).

and Todorovska [24–26]. Based on numerical simulations performed for a number of typical strong-motion instruments, Graizer [19] concluded that tilts could influence significantly the output of the horizontal pendulums. The effect of angular acceleration is significant for instruments with a long pendulum arm, as in the case of classical seismometers, but is small for typical accelerometers with a short pendulum arm. The effect of cross-axis sensitivity may reach few percents for motions higher than 2 g, and for accelerometers with a natural frequency of 25 Hz. Cross-axis sensitivity is almost negligible for modern accelerometers that have a natural frequency of about 100 Hz. The terms caused by tilting are always present for the horizontal penduli, and cannot be neglected.

For small oscillations, the vertical seismometer is almost not sensitive to tilts, and neglecting the cross-axis sensitivity terms the differential equations of the horizontal and vertical pendulums simplify to:

$$L : y_1'' + 2\omega_1 D_1 y_1' + \omega_1^2 y_1 = -x_1'' + g\psi_2 \quad (2a)$$

$$T : y_2'' + 2\omega_2 D_2 y_2' + \omega_2^2 y_2 = -x_2'' + g\psi_1 \quad (2b)$$

$$V : y_3'' + 2\omega_3 D_3 y_3' + \omega_3^2 y_3 = -x_3'' \quad (2c)$$

Thus, in a typical strong-motion triaxial instrument the two horizontal sensors are responding to the combination of inputs corresponding to horizontal accelerations and tilts, while the vertical sensor is mainly responding to the vertical acceleration. This may have important consequences, and raises the following questions when dealing with strong-motion records from earthquakes:

- To what extent is tilt responsible for the differences between horizontal and vertical components in long-periods during a real earthquake?
- Is there a principal difference in spectral content of horizontal and vertical motions, or does this difference mainly result from errors in recording horizontal accelerations which are contaminated with tilt?

The horizontal sensor (see Eqs. (2a) and (2b)) is sensitive to the second derivative of displacement and to tilt. This means that double integration of the Eq. (2a) or (2b) will produce the sum of displacement and double integrated tilt. Assuming that tilt is proportional to velocity [24], double integration will give results proportional to the integral of displacement, and the result can look like long-period noise.

Based on Eqs. (2a)–(2c) we suggest performing a simple test of tri-axial accelerograms: compare long-period components of the two horizontal and one vertical records. If their levels are of the same order, this can give us more confidence in attributing long periods to ground displacement. If the level of long-period motion is significantly higher in the horizontal components, this could possibly be due to tilts.

3. Residual displacements and what can be done in absence of recorded rotations (tilts)

Consider the differential equation of pendulum motion in absence of rotations. In this case Eqs. (2a) and (2b) can be simplified and will be similar to the Eq. (2c) for both vertical and horizontal components

$$y'' + 2\omega D y' + \omega^2 y = -V x'' \quad (3)$$

where V represents a magnification factor. The ground displacement $x(t)$ can be found by integrating recorded output of the instrument $y(t)$.

The first algorithm for computation of residual ground displacements from recorded strong motion accelerograms appears to have been given by Bogdanov and Graizer [4], and later modified by Graizer [5]. A key part of the proposed method involves baseline correction that can be accomplished by minimizing the functional (W), which is based on realistic assumptions about minimum velocity at the beginning and at the end of an earthquake ground motion

$$W = \int_0^{T_1} [x'(t)]^2 dt + \int_{T_2}^T [x'(t)]^2 dt \quad (4)$$

where T is the length of the recorded signal and T_1 and T_2 are times such that $0 < T_1 < T_2 < T$. This approach is based on the assumption that time intervals $[0, T_1]$ and $[T_2, T]$ can be found during which the ground motion is small compared to the strong motion amplitudes [5]. A baseline was first approximated by polynomials of up to the third degree, and later up to the higher degrees. In real applications of this method, polynomials of the second to fifth degrees were used for baseline correction.

The challenging part was to convince the seismological community that it is possible to recover residual displacement from records of real accelerographs. The first series of tests were performed with the Soviet film accelerograph SSRZ (very similar to the SMA-1). The instruments were placed on a shake-table (or a specially designed cart), and their permanent displacements were recorded independently by a ruler or a special gauge. Results of the tests were published in a series of papers [4,5] and reports. This is the most convincing classical way of testing the methods of data processing. The shake-table tests proved that residual displacement could be recovered from the record. Later, similar tests were performed in 1991 at the Institut de Physique du Globe (Strasbourg, France) and in 1993 with the Kinematics FBA, Terra Tech SSA-302, Sundstrand SSD3 and Teledyne SA-220 sensors in cooperation with W. Lee of the US Geological Survey in Menlo Park (1993).

During the first series of experiments performed at the end of the 1970s, we ‘discovered’ [4,5] that if the instrument is even slightly tilted during its movement, it makes recovery of permanent displacement almost impossible. First, a series of tests were simply performed by sliding the accelerograph along the surface of the table. In this case, to

overcome the static friction at the beginning of the motion, tilting of the instrument may occur. To avoid this problem a cart or shake-table was used in later tests [5,19,20]. Comparisons were also made of results obtained from different instruments installed at the same place, for example, from accelerograph and seismograph [5].

Another group of analyses was performed by numerical testing of the algorithm. In this case the ideal (calculated) response of the instrument was distorted by systematic and random errors. This group of tests is valuable because it

allows one to study the effects of each factor separately. It also allowed formulation of the requirements for the quality (dynamic range) of the records, necessary to obtain permanent displacements. These tests also showed that random errors in acceleration can result in long-period disturbance after double integration [5]. In this set of tests it was assumed that random errors have normal distribution with zero mean [27]. Double integrated random noise may result in additional errors in permanent displacement calculations. Similar to this result, Boore [28] concluded

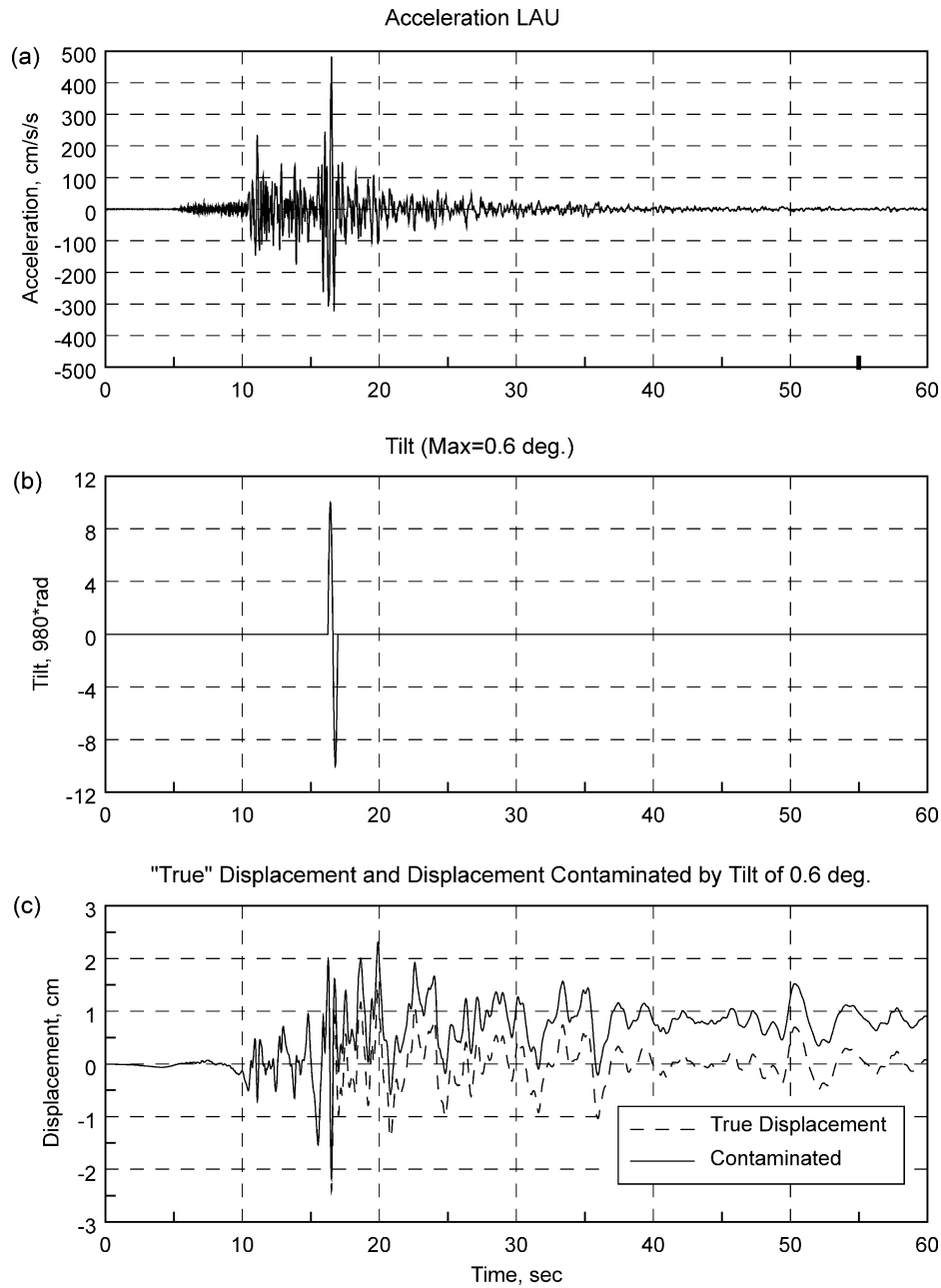


Fig. 2. Comparison of the 'true' displacement and displacement calculated using accelerogram contaminated by simple tilt: (a) test acceleration, (b) tilt with maximum amplitude of 0.6° , (c) true displacement calculated from the test acceleration (dashed line) and displacement calculated from the record contaminated by tilt (full line).

that analog-to-digital conversion of the signal can introduce significant drifts in displacements derived from digitally recorded accelerations.

The results of all these tests lead to a conclusion that processing real accelerograms, to get true ground displacement (including permanent displacement) requires the following conditions to be satisfied:

1. The input ground motion must be purely translational, without any tilting or any other natural distortions.

2. The record must contain clear beginning and ending parts with relatively small amplitudes to allow the baseline correction.
3. The signal to noise level of the record must be high enough, at least 40 dB.

Compliance with the second and third conditions is usually possible (and can be verified), especially for digital records. But the first condition cannot be verified unless independent measurements of rotations are performed.

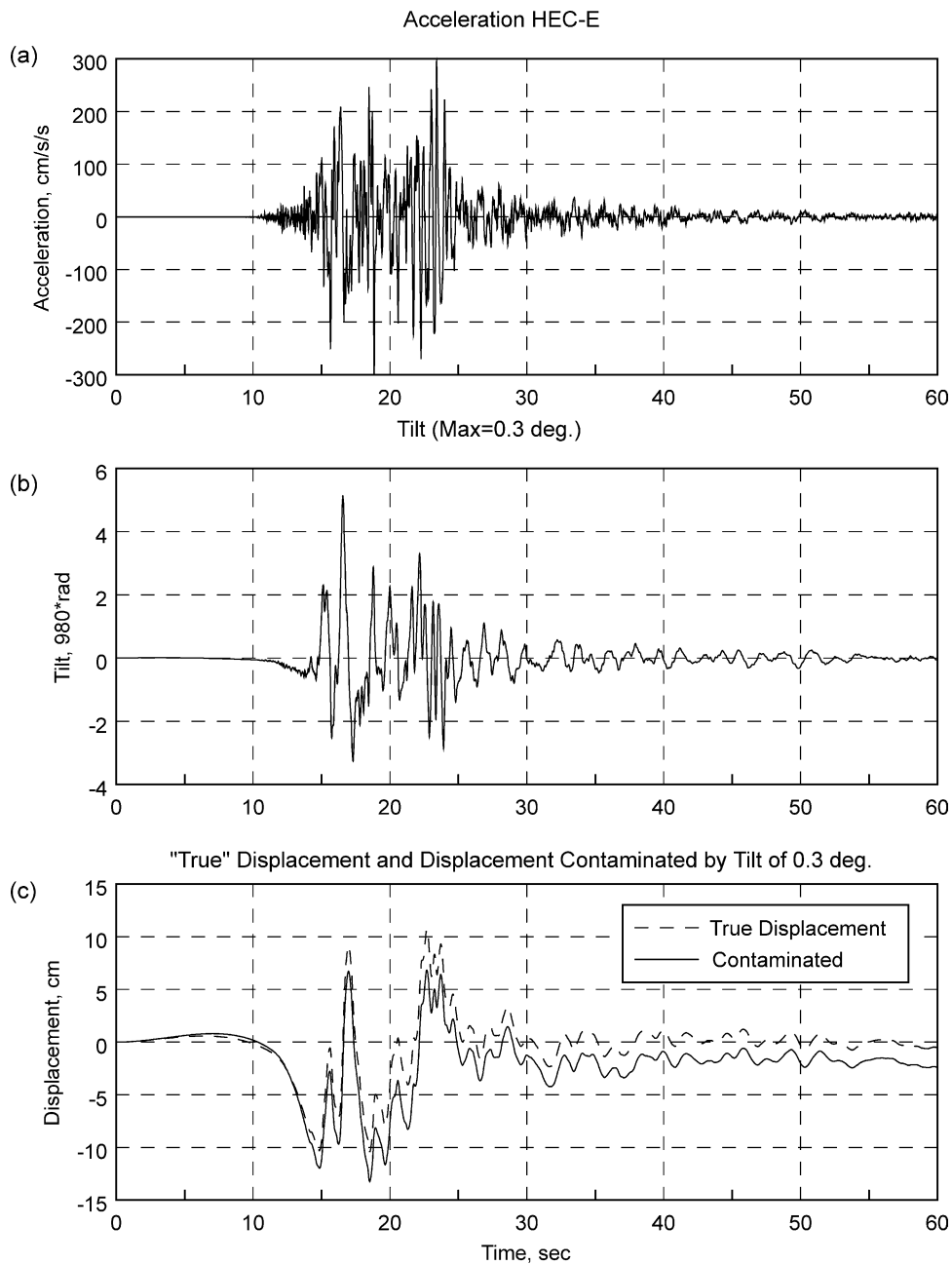


Fig. 3. Comparison of the 'true' displacement and displacement calculated using accelerogram contaminated by 'realistic' tilt: (a) test acceleration, (b) tilt with maximum amplitude of 0.3° , (c) true displacement calculated from the test acceleration (dashed line) and displacement calculated from the record contaminated by tilt (full line).

4. Numerical tests of the effects of tilt on computations of displacement

Tests were performed to study the influence of tilt on the ability of numerical algorithms to compute displacements, including permanent displacement. Fig. 2 shows the first test in which the acceleration record was ‘contaminated’ by tilt. The record was produced by correcting the accelerogram of the Northridge earthquake, recorded at Los Angeles—University Hospital Grounds (LAU). The test record was processed using the standard CSMIP procedure of filtering

(same as the procedure of Trifunac [1]). This test acceleration record does not have any long periods, and when integrated twice does not produce permanent displacement (Fig. 2c, dashed line).

The ideal test acceleration record (Fig. 2a) was contaminated by tilt record shown in Fig. 2b. The maximum amplitude of tilt was 0.6° , and had a simple shape of one period of a sinusoid (with a period of ~ 0.7 s). The maximum amplitude of acceleration resulting from tilt alone is about 2% of the peak translational acceleration. The displacement calculated by double integration of

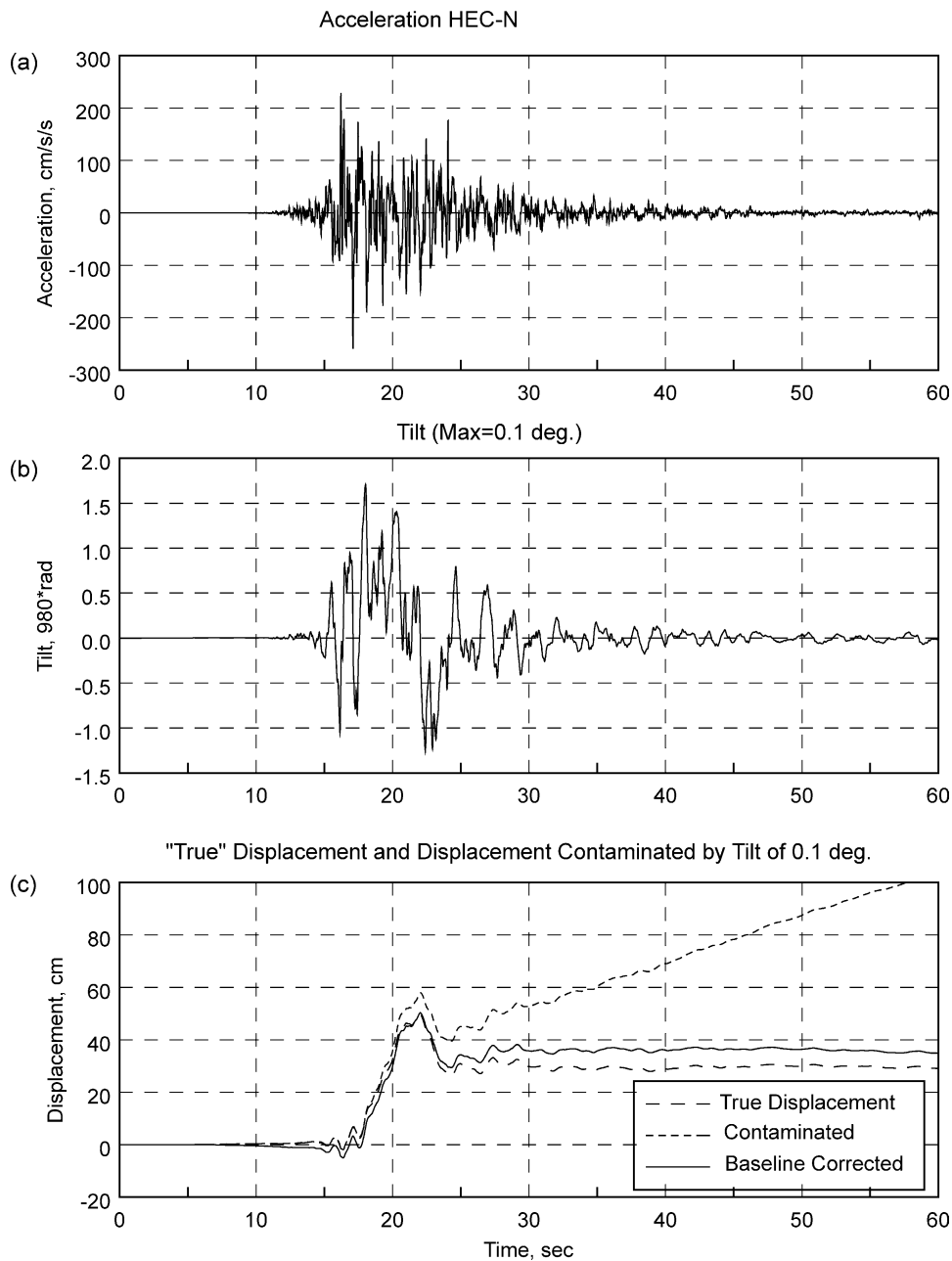


Fig. 4. Comparison of the ‘true’ displacement and displacement calculated using accelerogram contaminated by realistic tilt: (a) test acceleration, (b) tilt with maximum amplitude of 0.1° , (c) true displacement calculated from the test acceleration (dashed line) and displacement calculated from the record contaminated by tilt (dotted line), and displacement obtained using Graizer’s algorithm for baseline correction.

the acceleration record contaminated by tilt (ideal record + tilt record) produces displacements (Fig. 2c, full line) that look like a perfect case of displacement with permanent displacement.

In the second test a record that does not contain permanent displacements (Fig. 3c, dashed line) was also contaminated by tilt. This record was created by using the corrected displacement curve from the Hector station during the Hector Mine earthquake (E-W component, HEC-E). It was assumed that this specific solution is an ideal ground motion with the corresponding ideal acceleration shown in Fig. 3a.

The tilt record was generated based on the assumption that the tilt spectrum is proportional to the ground velocity spectrum [24]. The ground velocity curve was normalized to the maximum amplitude corresponding to the tilt of 0.3° . In this case maximum amplitude of tilt motion ($g\psi$) was about 1.7% of the peak translational acceleration. Fig. 3c then shows the two following curves: (1) true or ideal displacement (dashed line), and (2) displacement obtained by double integration of true acceleration contaminated by tilt (full line).

Results of these experiments (Figs. 2 and 3) show that short and long tilting of the instrument during an earthquake motion can result in what appears as permanent displacement of the ground. If tilting occurs during the strong motion it may look like a realistic permanent ground displacement.

The results of the third test are shown in Fig. 4. The record that contains permanent displacements (Fig. 4a) was contaminated by tilt. This record was created by using the corrected displacement curve from the Hector station during the Hector Mine earthquake (N-S component, HEC-N). It was assumed that this specific solution is an ideal ground motion with the corresponding ideal acceleration shown in Fig. 4a. As for the previous test, the tilt record was generated based on the assumption that the tilt spectrum is proportional to the ground velocity spectrum [24]. The ground velocity curve was normalized to the maximum amplitude corresponding to the tilt of 0.1° . In this case maximum amplitude of tilt motion ($g\psi$) was less than 1% of peak translational acceleration. Fig. 4c shows the following three curves: (3) true or ideal displacement (dashed line), (4) displacement obtained by double integration of true acceleration contaminated by tilt (dotted line), (5) displacement calculated using Graizer's algorithm for baseline correction [5] (full line). Application of the baseline correction algorithm [5] makes the displacement solution look very real. It produces an error of about 21% in permanent displacement calculation.

These tests examples are applicable to accelerograms from horizontal sensors. The results shown in Figs. 2–4 clearly demonstrate that tilt can contaminate results of ground motion calculations with what resembles permanent displacement. Their influence will result in non-reliable permanent displacement.

Tilt can also result in differences in the long-period component of the horizontal and vertical motions, since the vertical pendulum is much less sensitive to tilts than the horizontal ones. Thus, only records of vertical sensors (for tilts less than $\sim 10^\circ$) can be used for permanent displacement calculations.

The above results show that only a six-component accelerometer (measuring three translations and three rotations) or a three-component accelerometer in combination with gyroscopes (similar to those used in inertial navigation) allow reliable measurements of permanent displacements from recorded accelerograms. Coming back to the analysis of existing three component accelerograms, it is possible to conclude that conservative procedure developed by Trifunac and Lee [1,2] and other similar ones are the only way for routine processing of existing strong-motion data.

5. Conclusions

Analysis of the response of pendular accelerometers to complex input motion that includes translational and rotational components was performed. It is shown that even for small oscillations pendulum is sensitive to the translational acceleration, angular acceleration, cross-axis motion and tilt.

Strong-motion instruments which are used in seismological and earthquake engineering measurements, are sensitive not only to the translational motion, but also to tilt. This sensitivity can be neglected in some far-field measurements, but must be included in the near-field studies. Numerical experiments demonstrate that ignoring the tilt effects in strong-motion studies can introduce long-period errors, especially for calculation of residual displacements. In contrast to horizontal sensors, vertical sensors are less sensitive to tilt. This makes them potentially more usable for the long-period and residual displacement calculations.

Conservative methods of strong-motion data processing that involve filtering in a limited frequency band have a clear advantage, especially for routine data processing, because digital filters can eliminate the long-period components partially introduced by tilting.

Thus it is desirable to start measuring the rotational components of the strong-ground motion in combination with measurements of translational motion in the vicinity of active faults.

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